Weyl Sequence in a Computer: Bit Clocks That Add Up

1. One simple rule

Take a w-bit integer counter and **add the same number a** at every step, wrapping around at 2^w:

$x_{n+1}=(x_n+a) \bmod 2^w$

If you divide by 2^w you get a value in [0,1); that's the digital Weyl sequence.

2. Addition is linear—even bit by bit

Binary addition works one bit at a time with carries, but it is still *linear*:
The contribution of each bit of the increment travels through the bits of the counter without disturbing the others.
You can think of *each bit* of the step constant **a** as starting its own independent "clock."

3. Each set bit drives a saw-tooth

Suppose bit k of a is 1.
That means "add 2^k" at every step.
Look only at bit k of the running counter:

- It rises steadily (in binary, toggles) until it completes a full cycle,
- then drops back to zero,
- with period 2^{w-k}.

Viewed as a scaled number, that's a clean **saw-tooth wave**.

4. Coexisting waves

Every set bit of a produces its own saw-tooth:

- higher-order bits give fast ramps.
- lower-order bits give slow ramps.

Because addition is linear, these waves simply **add together**.
They never interfere or distort each other—the total value is just the sum of their contributions.

5. The whole sequence

The computer's Weyl output

$x_n/2^w$

is exactly the **superposition of those independent saw-tooth clocks**.
Change the increment a, and you change which clocks are present and how fast they run.

6. Why that matters

- This picture explains the uniform spread of values: the combined clocks visit every allowable state in turn.

- It also predicts the overall period:
  Period = 2^w / gcd(a, 2^w), which is just the least common multiple of the individual bit-clock periods. Or you can write Period = 2^{w-k}, where k is the index of the lowest set bit of a.
- If a is odd then all the possible binary states are visited.

Take-home idea:
A digital Weyl generator is nothing mysterious—just a set of independent saw-tooth clocks, one for every '1' in the step size.
Because addition is perfectly linear, those clocks coexist peacefully, and their sum is the sequence you see.